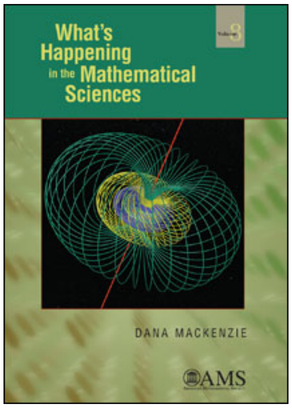
What's happening in the mathematical sciences, vol. 8 *Dana Mackenzie*, American Mathematical Society, 2011 (129 p.), soft cover, ISBN 978-0-8218-4999-6, US$23.00.



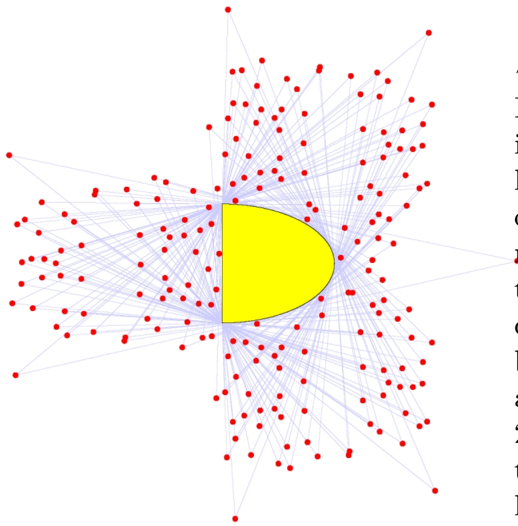
The series *What's Happening in the Mathematical Sciences* has been introduced in a previous review[1]. Dana Mackenzie has collected again nine very interesting texts on mathematical topics that have attracted recent attention in 2009-2010. I will introduce these topics very briefly.

We all do a lot of surfing on the Internet and we leave, willingly or unwillingly a lot of traces. Obviously we unveil ourselves on social sites like facebook, LinkedIn, Google+, etc. but we also twitter, blog, and e-shop our souls away. Even telephone traffic is traced, and, as some politicians can testify, things we definitely want to keep private may pop-up unexpectedly, much to the benefit of the tabloids and organizations like WikiLeaks. And there is profit to be made out of all this information just waiting to be harvested. *Accounting for taste*, the first article in this book, is reporting about the Netflix Prize competition. Netflix is an online movie service company. Like on Amazon and so many other sites, these companies have machine learning software following meticulously all your moves so that it can predict what kind of products you are interested in and hence what you are most inclined to buy so that they can be suggested to you. The competition was set up with the goal of improving the prediction made by their own software by 10%. After three years the winner was announced. It took the competitors a lot of hard work and the competition ended in a sprint with a very close call. The winner submitted the solution only 20 minutes before the second finalist. The article tells about the different types of prediction strategies and compares this with the Longitude Prize of the 18th century when the goal was to design a clock that could be taken out on a ship to determine the longitude when out at sea.

A brave new symplectic world is a text about symplectic topology. Like a simple pendulum on a spring (not a stiff rod), many phenomena in physics will result in chaotic dynamic behaviour. However, in 2008 Cliff Taubes (Harvard) proved that a two-parameter system like the pendulum will always have periodic solutions too, a conjecture formulated by Allan Weinstein (Berkeley) from the late 1970s. In this chapter, Mackenzie illustrates that results in a certain domain of physics or mathematics, often depends on results obtained in remote, seemingly unrelated areas of mathematics. It introduced the different results that were obtained by an interplay of topology and complex differential geometry, and that eventually resulted in the proof of Weinstein's conjecture. This proof gives an answer in the case of the 2-parametric system. It is still unclear what can be said in higher dimensions.

A topic that was certainly hot in the last couple of years and it still is, is the financial crisis. The bankruptcy of Lehman Brothers in September 2008, was a trigger of what we are still experiencing today. Has this been caused by the (obviously non-adequate) models that have been used by the traders? Hence should it be blamed on mathematics? In *Mathematics and the financial crisis*, the models used are illustrated along a time line from 2003 till 2009 and the corresponding financial evolution and the housing market in the USA. The 'formula that killed Wall Street' was the Gaussian copula formula. It inverts the Sklar formula giving some interdependencies of random variables when the marginal distributions are known. By inverting this, one could obtain the unknown marginal distributions (will the mortgage be paid off or not). When the model became under pressure, dependencies changed, assumed constants became variable and the Gaussian bell shape got tails heavier than usually assumed (extreme events become more usual). But one should never waste a good crisis. It can only lead to improving and refining the mathematics and models used.

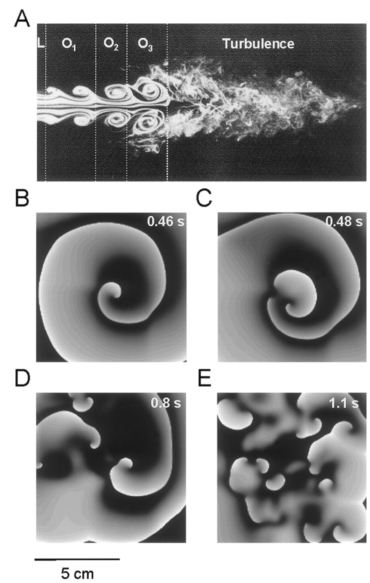
[1]BMS-NCM Newsletter, issue 79, September 2010.



outer billiard with semicircular hole

Do you know what an outer billiard is? It is explained in *The ultimate billiard shot.* The outer billiard game was proposed by Bernhard Neumann in 1959. It is infinitely large and has a hole in its center. The track of a ball runs in a straight line, tangent to the hole, from its current position to the tangent point and continues over the same distance. From that position, the previous step is repeated (using a different tangent) etc. The question is whether the billiard needs to be infinitely large. Usually not. So, the challenge becomes: is there a winning shot where the path of the ball spirals out to infinity? Rich Schwartz (Brown University) the author of *Outer billiards and kites*, (Princeton University Press, 2009) proved in 2007 that such a lucky shot (it is very sensitive to the starting point) is possible for certain convex polygonal holes like a diamond-like kite shape where the ratio between the short and the long rib is an irrational number. Results obtained for other cases are surveyed.

In 2009, there was a lot of political commotion about health insurance in the USA. In the midst of the debate some medical team advised that women between 40 and 49 should no longer be counseled to undergo a mammography. The USPTF[2] had to give advise on this issue. Previous recommendations were based on mathematical models with randomized control trials. For several reasons these were outdated and the task force decided to move to simulations of updated mathematical models instead. The story is told in *SimPatient.* Six different models were selected (5 from American institutes and one from The Erasmus Universiteit in The Netherlands). The paper where the results were published did not reveal everything necessary, to come to an undisputable conclusion. Thus the public advise not to have the mammography recommended between 40 and 49, is still being made on obscure grounds. For one thing, there is no mentioning of the 'cost' (in whatever units) of the screening. How much does it cost to add one quality year to a person's life? How about overdiagnosis? Is saving a couple of lives worth to do unnecessary operations on false diagnoses? In short, mathematical models may have gained a place in political discussions but they are still far from perfectly error-free. "All models are wrong, but some of them are useful" (George Box)



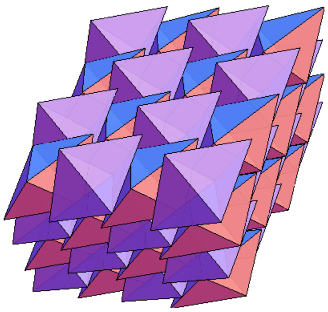
Transition to chaos

Instant randomness discusses the fact that the transition from (partially) ordered to completely random often appears very sudden. A. Peres conjectured in 1999 that this kind of cutoff between local deviation from randomness and its smoothed out version is a general feature of dynamical systems. This rapid phase transition is e.g. known in the Ising-Lenz model formulated in the 1920s. It can be shown by the Metropolis algorithm (1959, Nicolas Metropolis) that later evolved to Markov Chain Monte Carlo methods (MCMC). Mackenzie explains these methods explains how it was finally shown by E. Lubetzky and A. Sly in 2010 that such cutoff phenomena did indeed occur at some instance and with an abrupt transition.

It is well known that chaos is a nonlinear phenomenon, while quantum physics is depending on the Schödinger equation, which is linear. It has standing wave solutions corresponding to certain eigenvalues. In a plane, these can be visualized by curves where the solutions are zero. These can be compared with the path traced by a billiard ball if the domain is considered to be a friction-free billiard table with perfectly reflecting boundaries. What happens to these patterns when the eigenvalue tends to infinity? If it would become a uniform gray all over the considered domain (in the QUE context below the domain is a negatively curved manifold), there would be some kind of chaos. The quest traced in *In search of quantum chaos* is the search for a proof of the Quantum Unique Ergodicity (QUE) conjecture (P. Sarnak, Z. Rudnick early 1990) saying that this uniform grey is the unique limit behaviour. A general proof is still

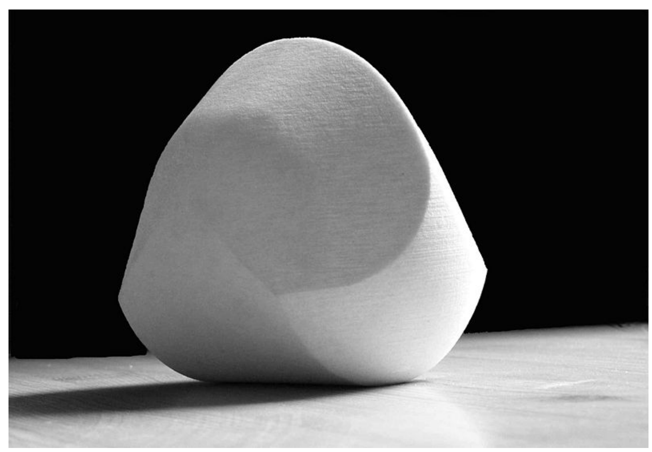
[2] US Preventive Task Force

unknown, but a particular case, the so called holomorphic QUE, is. Surprisingly this is closely connected with the Ramanujan conjecture (proved by the Belgian mathematician Pierre Deligne in 1974) and other related results in number theory. Using the machinery of Hecke operators and L-functions, available in number theory allowed to bypass the Riemann Hypothesis and to prove the holomorphic QUE in quantum theory. This gives some hope to tackle the general case.



Tetrahedral packing

In 1611 Kepler conjectured that the way oranges are stacked in a grocery store is the best (densest) way of sphere packing. It was only proved in 1998 that this is indeed the case filling up 74.05% of the three dimensional space. The 18th problem formulated by Hilbert in 1900 generalized this from spheres to other identical objects. In this context, a scientific race to design the densest possible packing with tetrahedra has been going on recently. Torquato and Conway found a non-lattice packing of tetrahedra with 71.75% density in 2006, making believe that tetrahedra were worse than spheres. However in 2008 B. Chen published a packing with 77.86% density, breaking the barrier of the spheres. From the field of quasicrystals, an even better packing of 85.03% popped up in a chemists group of S.C. Glotzer. As the packings became denser, the construction became more complex. However a team at Cornell used techniques of V. Elser, designed for constrained optimisation, and obtained in 2010 a very simple solution that fills 85.47% of space. It was promptly improved by Torquato to 85.55%, which seems to be the current record. This thrilling story is told in *3D surprises*, which also introduces the Gömböc, a 3D object that has exactly 2 stationary positions: one stable and one unstable and spontaneously takes on the stable one. It is self-righting and self-wronging.



A Gömböc

In the last chapter *As one heroic age ends, a new one begins*, another Belgian is featuring. The chapter is about a topological problem where one is looking for framed manifolds with Kervair invariant 1 in higher dimensions. What these concepts mean is first made clear for the non-topologist. Although the problem was formulated in the middle of the 20th century, so far one has only proved the existence of these objects in dimensions $n = 2^k - 2$, for $k = 2, \ldots, 8$. For $n = 62$ it was found in 1984, and to the surprise of many, it was proved by D.C. Ravenel et al. that these objects did not exist for $n = 254$ and up. Whether they exist for $n = 126$ is still an open problem. The Belgian involved here is Jos Leys who produced the picture that is shown on the cover of the book, depicting a Hopf fibration. It consists of a set of circles that are linked to all other circles in the sense that one cannot separate them without cutting one of the circles. It appears in one of the construction schemes of the above problem. These circles are pre-images that are then mapped onto points on an ordinary 2-dimensional sphere from which a proof can be concluded. Jos Leys is an independent graphic designer living in the Antwerp area who may be known from his collaboration in producing the 2 hour animation movie *Dimensions*[3] and you may enjoy many of his mathematically oriented imaginary on his website galleries `www.josleys.com`.

This volume 8 in the series is another set of fascinating windows on diverse actual topics in mathematics. It is written for mathematicians, but one need not be a specialist in the topic that is being discussed. Everything is perfectly made clear and it should be an eye-opener for many of us who have dug themselves too deeply in their own niche so that they miss all the fun and excitement that is going on in completely different areas. I can only mention one weak point: the references to the illustrations are not always correct, (and there are many illustrations, sometimes several pages before or after the reference) so that the reader has to thumb back and forth to find the matching picture.

Adhemar Bultheel

[3] See `http://www.dimensions-math.org/`