PADÉ TECHNIQUES FOR MODEL REDUCTION IN LINEAR SYSTEM THEORY : A SURVEY

A. Bultheel and M. Van Barel (★)

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Techniques of Padé approximation and continued fractions have been used often in model reduction problems. An extensive bibliography on this topic is given. The ideas are explained for the simple situation of a scalar function where no singular blocks appear in the Padé table. Extensions to the matrix case and the multivariable case are not explained in detail.

Keywords : Model reduction, continued fractions, Padé approximation.

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(★) K.U.Leuven, Dept. Computer Science, Celestijnenlaan 200A, B-3030 Leuven (Heverlee), Belgium.

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1. Introduction

In linear system theory, the problem of rational approximation is an important topic. Techniques from Padé theory and continued fractions have been quite successful in this respect due to the linearity of the problem and the ease of computation. The problem of partial minimal realization has been recognized to be equivalent with Padé approximation. We compiled a bibliography on model reduction in a wide sense where Padé techniques and continued fractions are used. It became clear after a short while that a detailed description of the most general case is not possible in the frame of this paper. Therefore we content ourselves to settle the ideas by giving a survey of the literature in the simplest situation. This is for the case of a so-called normal Padé table in the scalar case. The non-normal case as well as the case of multivariable and multidimensional systems is only briefly mentioned. The main objective of the paper is to link the ideas of Padé approximation with certain problems in system theory. This is fairly easy for the simplified situation just mentioned. The Padé literature seems to be more mature in this situation as well as in the non-normal scalar case. However in the multivariable and multidimensional problems, system theory seems to be more advanced. It is our hope that this paper will raise mutual interest and that new results may grow out of it.

The structure of the paper is as follows : In the first four sections we recall some definitions from continued fractions and linear systems. The model reduction problem as defined in section five will then be elaborated on in subsequent sections. A basic tool is the Routh algorithm introduced in section 6. It is applied to model reduction in different situations in section 7. Contracted forms and evaluation algorithms are mentioned in sections 8 and 9. Stability is an important problem for systems. It is defined in section 10 and section 11 explains how continued fractions enter the picture in stability checking algorithms. Section 12 then shows how Padé type approximants may be constructed meeting this stability condition. Realization of a transferfunction by a digital or analog network and corresponding state space descriptions can be easily obtained from the coefficients in continued fractions as is shown in sections 13 and 14. The frequency response and power spectrum of a system are introduced in section 16 and Chebyshev-Padé techniques to approximate them are given in section 17. Mixed methods are presented in sections 18 and 19. A brief classification of the literature for non normal Padé tables, multi-input - multi-output systems and two-dimensional systems are given in the last three sections.

As we already mentioned, we compiled a bibliography on the subject where we confined ourselves to papers from system theory literature where Padé approximants and continued fractions were more or less explicitly used. Reference to these papers is given in the text by a number of letters followed by the year of publication. If papers are cited that are not related to system theory, we refer to them by a number. These references are compiled separately.

2. Continued fractions (CF)

Let F be a field to which an element ∞ is added. We suppose that for ∞, the usual arithmetic rules hold. With two sequences $\{a_n\}_0^\infty$ and $\{b_n\}_0^\infty$ of F-elements we associate a sequence of linear fractional transforms $\{t_n\}_0^\infty$ for F viz.

$$t_0(w) = \frac{a_0}{b_0} + w \text{ and } t_n(w) = \frac{a_n}{b_n + w}, \quad n = 1,2,\ldots \quad . \tag{2.1}$$

This sequence is called a continued fraction. It is denoted by

$$\frac{a_0}{b_0} + \frac{a_1}{\left| b_1 \right.} + \frac{a_2}{\left| b_2 \right.} + \ldots = \frac{a_0}{b_0} + \sum_{n=1}^{\infty} \frac{a_n}{\left| b_n \right.} . \tag{2.2}$$

$c_n = t_0 \circ t_1 \circ t_2 \circ \ldots \circ t_n(0)$ is called its nth convergent. There are different ways to evaluate the nth convergent. The backward evaluation scheme consists in evaluating $t_n(0)$, then $t_{n-1}(t_n(0))$ etc. until $c_n = t_0 \circ t_1 \circ \ldots \circ t_n(0)$. In the forward evaluation scheme one evaluates the numerator and denominator separately as follows

$$A_n = b_n A_{n-1} + a_n A_{n-2}$$

$$B_n = b_n B_{n-1} + a_n B_{n-2} \qquad n = 1,2,3,\ldots \text{ with } \begin{bmatrix} A_{-1} & A_0 \\ B_{-1} & B_0 \end{bmatrix} = \begin{bmatrix} b_0 & a_0 \\ 0 & b_0 \end{bmatrix}$$

and $c_n = A_n/B_n$.

Two CF's are called equivalent if they have the same convergents.

A CF equivalent to (2.2) has the form

$$\frac{a_0\,k_0}{b_0\,k_0} + \frac{a_1\,k_1}{\left| b_1\,k_1 \right.} + \sum_{n=2}^{\infty} \frac{k_{n-1}\,k_n\,a_n}{\left| k_n\,b_n \right.}\,, \quad k_n \neq 0\,, \quad n = 0,1,\ldots. \qquad (2.3)$$

Such an equivalence transform can be used to make some of the $k_{n-1}\,k_n\,a_n$ or the $k_n\,b_n$ equal to 1.

The <u>even part</u> or the <u>even contraction</u> of the CF (2.2) is a CF with convergents c_{2n}, $n = 0,1,\ldots.$
It is of the form

$$\frac{a_0}{b_0} + \frac{a_1\,b_2}{\left| a_2 + b_1\,b_2 \right.} + \sum_{n=2}^{\infty} \frac{-a_{2n-2}\,b_{2n-2}^{-1}\,a_{2n-1}\,b_{2n}}{\left| a_{2n} + (b_{2n-1} + b_{2n-2}^{-1}\,a_{2n-1})\,b_{2n} \right.}\,, \quad b_{2n} \neq 0$$

or some equivalent form.

The <u>odd part</u> is defined in a similar way.

A recent reference on continued fractions is [4].

3. <u>Division algorithms</u>

Suppose H(z) is given as the ratio of two functions

$$H(z) = \frac{S_0(z)}{S_{-1}(z)}\,.$$

We can construct a formal continued fraction expansion of this function as follows

$$H(z) = \frac{1}{S_{-1}(z)/S_0(z)} = \frac{1}{\left| b_1(z) \right.} + \frac{a_2(z)}{\left| S_0(z)/S_1(z) \right.}\,, \text{ with}$$

$$S_1 = (S_{-1} - b_1\,S_0)/a_2\,,$$

$$= \frac{1}{\left| b_1(z) \right.} + \frac{a_2(z)}{\left| b_2(z) \right.} + \frac{a_3(z)}{\left| S_2(z)/S_1(z) \right.}\,, \text{ with}$$

$$S_2 = (S_0 - b_2\,S_1)/a_3$$

etc.

It is easy to verify that if the nth convergent has the form P_n/Q_n, then

$$Q_n H - P_n = (-1)^n a_2 \dots a_{n+1} S_n/S_{-1} . \qquad (3.1)$$

The $b_k(z)$ and $a_{k+1}(z)$ could be chosen such that the computation

$$S_k = (S_{k-2} - b_k S_{k-1})/a_{k+1} \qquad (3.2)$$

introduces no pole in S_k.
As an example we can take for some complex number α: $b_k = S_{k-2}(\alpha)/S_{k-1}(\alpha)$ and $a_{k+1}(z) = z - \alpha$. It is clear from (3.1) that under certain normality conditions the nth convergent will interpolate at $z = \alpha$.
Another example is to take for b_k the linear interpolating polynomial at α and β (where eventually $\alpha = \beta$). a_{k+1} will then be $(z-\alpha)(z-\beta)$. In this way we could construct a continued fraction which is just a contraction of the one constructed in the previous example.

4. Linear systems

We shall consider linear, lumped, time-invariant, causal and assymptotically stable (see section 10) systems. They are supposed to be single-input - single-output (SISO) in the main part of this paper. There are different ways to characterise a system. It may be given by its state space description :

$$S\,x(t) = A\,x(t) + B\,u(t) \quad , \quad A \in \mathbb{C}^{n \times n} \; , \; B \in \mathbb{C}^{n \times 1}$$

$$y(t) = C\,x(t) \qquad\qquad C \in \mathbb{C}^{1 \times n}$$

$$x(0) = 0 \quad , \quad t \in T$$

u is the input, y is the output and x is the state vector. For a discrete time (d.t.) system is $T = \{0,1,2,\dots\}$ and the operator S represents the shift operator $S\,x(t) = x(t+1)$. For a continuous time (c.t.) system is $T = [0,\infty)$ and S stands for differentiation $S\,x(t) = \dot{x}(t)$.
Another possible description of the system is its transfer function

$$H(z) = C(zI - A)^{-1}B.$$

It is a strictly proper rational function. This function depends on the frequency domain variable z. It is the Laplace transform in continuous time or the discrete Laplace transform (i.e. z-transform) in discrete time of the impulse response $h(t)$, $t \in T$.

$$H(z) = \sum_{t=1}^{\infty} h(t)\, z^{-t} \quad ; \quad h(t) = \frac{1}{2\pi} \int_0^{2\pi} e^{i\omega t}\, H(e^{i\omega})\, d\omega \quad , \quad \text{(d.t.)},$$

$$H(z) = \int_0^{\infty} e^{-zt}\, h(t)\, dt \quad ; \quad h(t) = \int_{-\infty}^{\infty} e^{i\omega t}\, H(i\omega)\, d\omega \quad , \quad \text{(c.t.)}.$$

The Markov parameters are defined by

$$m_k = \frac{1}{k!} \frac{d^k H(z)}{dz^k}\bigg|_{z=\infty} = C\, A^{k-1}\, B \, , \quad k = 1,2,\ldots$$

$$= h(k) \qquad \text{(d.t.)} \, ,$$

$$= \frac{d^{k-1} h(t)}{dt^{k-1}}\bigg|_{t=0} \qquad \text{(c.t.)}.$$

The numbers $C\, A^k\, B$, $k < 0$ are related to time moments.
These are defined for $k = 0,1,2,\ldots$ by

$$M_k = \int_0^{\infty} t^k\, h(t)\, dt = (-1)^k \frac{d^k H(z)}{dz^k}\bigg|_{z=0} = (-1)^{k+1} k!\, C\, A^{-(k+1)}\, B \quad \text{(c.t.)}$$

and

$$M_k = \sum_{t=0}^{\infty} t^k\, h(t) = (-1)^k \left(z \frac{d}{dz}\right)^k H(z)\bigg|_{z=1} \quad \text{for} \quad \text{d.t.} \, .$$

The relation between M_k and $C\, A^{-k}\, B$ in d.t. is not direct. It is however clear that M_k can be written as a linear combination of the numbers

$$B_k = (-1)^k \frac{d^k H(z)}{dz^k}\Big|_{z=1} = (-1)^{k+1} k!\, C(A-I)^{-(k+1)} B.$$

[HWAb81].

If a transfer function is given as a rational function

$$H(z) = \frac{c_0 + c_1 z + \ldots + c_{n-1} z^{n-1}}{d_0 + d_1 z + \ldots + d_{n-1} z^{n-1} + z^n},$$

we can immediately write a state space form for the system as

$$S\,x = A\,x + B\,u$$
$$y = C\,x$$

with $B^T = [0 \ldots 0\ 1]$, $C = [c_0\ c_1 \ldots c_{n-1}]$ and A the companion matrix of the denominator of H(z) :

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \ldots & 0 & 1 \\ -d_0 & -d_1 & -d_2 & \ldots & -d_{n-1} & -d_{n-1} \end{bmatrix}.$$

This state space realization is called the phase variable canonical form. If K is a nonsingular matrix, then equivalent state space equations are of the form

$$S\,v = A'v + B'u$$
$$y = C'v$$

with $v = K\,x$, $A' = K\,A\,K^{-1}$, $B' = K\,B$, $C' = C\,K^{-1}$, K nonsingular.

5. The Model reduction problem

We shall consider the problem of model reduction to be defined as follows : Given some information about a system with a rational transfer function (of a high degree), find a system with a lower degree rational transfer

function that in some sense approximates the original system. The information given could be a state space description, Markov parameters, time moments, or frequency information like frequency response, numerator and denominator of the transfer function, the autocorrelation function of the impulse response or its energy spectrum. The form in which the approximating system is obtained can also take different forms.

6. The Routh algorithm

$$\text{Suppose } H(z) = \sum_{n=0}^{\infty} c_n \zeta^n \Big/ \sum_{n=0}^{\infty} d_n \zeta^n \quad , \quad d_0 \neq 0$$

$$= \sum_{0}^{\infty} f_n \zeta^n \qquad (f_{-n} = 0 \ , \ n = 1,2,\ldots)$$

in a neighborhood of $\zeta = 0$. Suppose H is normal, i.e. $\det(f_{m+i-j})_{i,j=0}^{n} \neq 0$ for all m and n. Then the Routh algorithm is a division algorithm that tries to interpolate at $\zeta = 0$ in each step. It can be formulated as follows

$$\left\lfloor \begin{array}{l} c_k^{(-1)} = d_k \quad , \quad k = 0,1,2,\ldots \\ \\ c_k^{(0)} = c_k \quad , \quad k = 0,1,2,\ldots \\ \\ \text{for } n = 0,1,2,\ldots \\ \\ \left\lfloor \begin{array}{l} b_{n+1} = c_0^{(n-1)} / c_0^{(n)} \\ \\ c_k^{(n+1)} = c_{k+1}^{(n-1)} - b_{n+1}\, c_{k+1}^{(n)} \quad , \quad k = 0,1,2,\ldots \end{array} \right. \end{array} \right. \qquad (6.1)$$

With the numbers b_k found by this algorithm we can construct the continued fraction of second Cauer form

$$\cfrac{1}{b_1} + \cfrac{\zeta}{b_2} + \cfrac{\zeta}{b_3} + \cfrac{\zeta}{b_4} + \ldots \qquad (6.2)$$

Concerning this continued fraction we have the following

Theorem

The rth convergent $H_r(\zeta)$ of (6.2) has the following properties

- It is irreducible.
- The numerator degree is $k = [(r-1)/2]$ and the denominator degree is $\ell = [r/2]$.
- It is the (k,ℓ) Padé approximant of $H(z)$. I.e.

$$H(\zeta) - H_r(\zeta) = O(\zeta^r), \qquad \text{for } \zeta \to 0. \tag{6.3}$$

□

The inspection of the degrees of the convergents shows that this algorithm computes a descending staircase in a normal Padé table. It is a simple matter to derive slightly modified versions of this algorithm to produce a continued fraction that is equivalent with the previous one [JEL79] and has the same properties. It may e.g. have the form

$$\cfrac{a_1}{\left|1\right.} + \cfrac{\zeta a_2}{\left|1\right.} + \cfrac{\zeta a_3}{\left|1\right.} + \cfrac{\zeta a_4}{\left|1\right.} + \dots . \tag{6.4}$$

7. Application of the Routh algorithm to model reduction

7.1 Modelling the steady state behaviour

This type of approximation is very popular and a great number of references are concerned with it:
[BOSL72,BOSLa73,BOSLb73,BROWa71,BROWb71,CAL74,CHE68,CHE69,CHE70, CHEa74,CHU75,GIB69,HAD83,HUA76,KHAa75,KHAb76,KHAb77,LALa74,LALb74, LAM82,LEE71,PALa80,PALc80,PAY56,SHAb74,SHIa71,SHIb71,SHIa74,SHId74, SHIb81,SHIH73,SHIH75,SHIH78,ZAK73].

Suppose α is some complex number and let $\zeta = z-\alpha$. Suppose that the transfer function $H(z)$ is given as a rational function and brought into the form

$$H(\zeta) = \frac{c_0+c_1\zeta+c_2\zeta^2+\dots+c_{n-1}\zeta^{n-1}}{d_0+d_1\zeta+d_2\zeta^2+\dots+d_n\zeta^n} . \tag{7.1}$$

Apply the Routh algorithm to this function to construct a continued fraction of the Cauer second form (6.2). We can ask if α could be chosen to give it a system theoretic meaning. Here we have to distinguish between d.t. and c.t. systems. For c.t. systems we can

set $\alpha=0$. It follows from the definitions of the time moments that near $z=0$ we have

$$H(z) = M_0 - M_1 z + M_2 \frac{z^2}{2!} - M_3 \frac{z^3}{3!} + \ldots \qquad (7.2)$$

where M_k are the c.t. time moments.
This means that the rth convergent of (6.2) will represent a transfer function of a reduced system that matches the time moments M_k, $k=0,1,\ldots,r-1$. I.e. it models the steady state behaviour.
For a d.t. system, the situation is somewhat different. Take in this case $\alpha=1$. We know that for z near 1 we have

$$H(z) = B_0 - B_1(z-1) + B_2(z-1)^2 - B_3(z-1)^3 + \ldots . \qquad (7.3)$$

Thus here the reduced system will have a transfer function with an expansion around $z=1$ that matches the first r terms of (7.3). But since M_k, $k=0,1,\ldots,r-1$ can be found as a linear combination of the numbers B_k, $k=0,1,\ldots,r-1$, it is clear that also the first r time moments are matched in the d.t. case. Thus again the steady state response is modelled.
If the reduced system is not asymptotically stable (see section 10), the correspondence is purely formal since its time moments don't even exist (see section 4).

If the system is not given by its transfer function, but by its time moments, its impulse response or its state space description, we can always find the series expansion of $H(\zeta)$ around $\zeta=0$. I.e. the expansions (7.2) or (7.3). The Routh algorithm can then be applied with this series as numerator and the constant 1 as denominator.

7.2 Modelling the transient behaviour

This problem is equivalent with the partial realization problem. Many of the papers in partial realization deal with the non-normal case or the multivariable case. They are not included here [CHE69,SHId74,SHIc81].
To find a Padé approximant around $z=\infty$, we can write the transfer function as

$$H(z) = \frac{1}{z}\left(\frac{c_{n-1}+c_{n-2}z^{-1}+\ldots+c_0 z^{-(n-1)}}{d_n+d_{n-1}z^{-1}+\ldots+d_0 z^{-n}}\right). \qquad (7.4)$$

The Routh algorithm starting with $c_k^{(-1)}=d_{n-k}$, $k=0,1,\ldots,n$ and $c_k^{(0)}=c_{n-1-k}$, $k=0,1,\ldots,n-1$ will give a continued fraction of the form

$$\cfrac{1}{zb_1} + \cfrac{1}{b_2} + \cfrac{1}{zb_2} + \cfrac{1}{b_4} + \cfrac{1}{zb_5} + \ldots \qquad (7.5)$$

which is called the first Cauer form.
It is easily found from the theorem given before that we have now the next

Theorem

The rth convergent $H_r(z)$ of this continued fraction (7.5) has the following properties

- It is irreducible.
- The numerator is a polynomial in z^{-1} of degree $k=[(r+1)/2]$ and it vanishes at $z=\infty$.
  The denominator is a polynomial in z^{-1} of degree $\ell=[r/2]$.
- It is the (k,ℓ) Padé approximant of $H(z)$ in $z=\infty$, I.e.

$$H(z) - H_r(z) = O(z^{-(r+1)}) \qquad \text{for } z \to \infty.$$

□

Since in this case $(\frac{d}{dz})^k H(z)\Big|_{z=\infty} = (\frac{d}{dz})^k H_r(z)\Big|_{z=\infty}$, $k=0,1,\ldots,r$,

we have matched the Markov parameters m_k, $k=1,\ldots,r+1$ I.e. the impulse response in the neighbourhood of $t=0$ is matched. Thus if $h(t)$ and $h_r(t)$ are the impulse response of the original and of the reduced system respectively, then

$$h(t) = h_r(t), \qquad t=1,\ldots,r+1 \qquad \text{(d.t.)}$$

$$(\frac{d}{dt})^k h(t) = (\frac{d}{dt})^k h_r(t), \qquad k=1,\ldots,r+1 \qquad \text{(c.t.)}.$$

By construction we have $h(0) = h_r(0) = 0$. This clearly shows that we model the transient behaviour of the system. If the Markov parameters themselves are given we can write $H(z)$ as

$$H(z) = \frac{1}{z} \cfrac{\Sigma_1^\infty k!m_k z^{-(k-1)}}{1}$$

and again apply the Routh algorithm.

If the state space description is given, we first have to compute $m_k = CA^{k-1}B$, $k=1,2,\ldots$ before the Routh algorithm is applicable.

7.3 Simultaneous modelling of steady state and transient behaviour

The method of time moments concentrated on the steady state response of the system. The following references treat the modelling of both transient and steady state behaviour. The basic idea is explained below.

[ASb82,BISb80,BIS81,CHU70,CHUa73,CHEb73,DAL79,GOL77,GOL81,KHA80, MARa83,PART77,PARTa78,PARTb78,PARTa79,PARTd82,PARTa83,PARTb83, SHAb74,SHAa76,SHAa79,VAR79]

Suppose as before that the transfer function is brought into the form (7.1) with $\zeta=z-\alpha$ and $\alpha=1$ for d.t. and $\alpha=0$ for c.t. To model both steady state and transient behaviour, we can alternate between the previous strategies. This gives the following algorithm :

$$c_k^{(-1)} = d_k \quad , \quad k=0,1,\ldots,n$$

$$c_k^{(0)} = c_k \quad , \quad k=0,1,\ldots,n-1; \; c_n^{(0)} = 0$$

for $k=0,1,2,\ldots,n-1$

$$b_{2k+1} = c_0^{(2k)} / c_0^{(2k-1)}$$

$$c_i^{(2k+1)} = c_{i+1}^{(2k)} - b_{2k+1}\, c_{i+1}^{(2k-1)} \quad , \quad i=0,1,\ldots,n-k-1$$

$$b_{2k} = c_{n-i-1}^{(2k+1)} / c_{n-i-1}^{(2k)}$$

$$c_i^{(2k+2)} = c_i^{(2k+1)} - b_{2k}\, c_i^{(2k)} \quad , \quad i=0,1,\ldots,n-k-2.$$

This algorihtm constructs a continued fraction of the form

$$\cfrac{1}{b_1} + \cfrac{\zeta}{b_2} + \cfrac{1}{b_3} + \cfrac{\zeta}{b_4} + \ldots \; . \qquad (7.6)$$

This is the modified Cauer form.
Under certain normality conditions we have now

Theorem

The rth convergent $H_r(\zeta)$ of continued fraction (7.6) has the following properties

- The numerator is a polynomial in ζ of degree $k=[(r-1)/2]$ and the

denominator is a polynomial in ζ of degree $\ell=[r/2]$.

- It is a two-point Padé approximant for $H(\zeta)$ in the sense that

$$H(\zeta) - H_r(\zeta) = O(\zeta^{k+1}) \qquad \text{for } \zeta \to 0$$

and

$$H(\zeta) - H_r(\zeta) = O(\zeta^{-(\ell+1)}) \quad \text{for } \zeta \to \infty.$$

□

Note that by alternating between the approximations at $\zeta=0$ and $\zeta=\infty$ we have a balancing between the modelling of steady state and transient. There is no reason why this should be alternating between these two points. We could also match p time moments first and then q Markov parameters or take any other combination [LUc83].

7.4 Approximation at z=α

In previous approaches we always approximated at z=0 (=1) and/or z=∞. There may be reasons that one should prefer to include some general point α, eventually in combination with z=0 (z=1) and/or z=∞. The algorithms are as before if the appropriate transformation is made.
For a c.t. system, an approximant with r interpolation conditions in z=α will match the moments

$$\left.\frac{d^k H(z)}{dz^k}\right|_{z=\alpha} = (-1)^k \int_0^\infty e^{-\alpha t}\, t^k g(t)\, dt \qquad , k=0,1,\ldots,r-1.$$

Such types of approximation were used in [AUDa77,AUDb77,DAV74,LUb83, PALa82,PARTb75,PARTb82].

8. Contracted forms

As we mentioned in connection with division algorithms, it is possible to modify the previous Routh type algorithms to construct contracted forms of the previously mentioned continued fractions directly. The second Cauer form can be contracted to a continued fraction of the form [KHAa76]

$$\cfrac{1}{a_1+b_1\zeta} + \cfrac{\zeta^2}{a_2+b_2\zeta} + \cfrac{\zeta^2}{a_3+b_3\zeta} + \ldots . \qquad (8.1)$$

It is constructed by the algorithm

$$c_k^{(-1)} = d_k \quad , \quad k = 0,1,\ldots$$

$$c_k^{(0)} = c_k \quad , \quad k = 0,1,\ldots$$

for $n = 0,1,2,\ldots$

$$a_{n+1} = c_0^{(n-1)} / c_0^{(n)}$$

$$c_k^{(n+1)} = c_{k+1}^{(n-1)} - a_{n+1}\, c_{k+1}^{(n)} \quad , \quad k = 0,1,\ldots$$

$$b_{n+1} = c_0^{(n+1)} / c_0^{(n)}$$

$$c_k^{(n+1)} = c_{k+1}^{(n+1)} - b_{n+1}\, c_{k+1}^{(n)} \quad , \quad k = 0,1,\ldots$$

A similar contraction can be obtained for the first Cauer form [KHAa76 SHI280] which gives

$$\left|\frac{1}{a_1\, z+b_1}\right| + \left|\frac{1}{a_2\, z+b_2}\right| + \left|\frac{1}{a_3\, z+b_3}\right| + \ldots \quad . \tag{8.2}$$

The contraction of the modified Cauer form gives the <u>mixed Cauer form</u> or <u>third Cauer form</u> [GOL81, HWAa82, KHA78, SHIb74, SHIc74, SHId74, SHIe74, SHIe75]

$$\left|\frac{1}{a_1+b_1\zeta}\right| + \left|\frac{\zeta}{a_2+b_2\zeta}\right| + \left|\frac{\zeta}{a_3+b_3\zeta}\right| + \ldots \quad . \tag{8.3}$$

It is constructed by the algorithm

$$c_k^{(-1)} = d_k \quad , \quad k = 0,1,\ldots,n$$

$$c_k^{(0)} = c_k \quad , \quad k = 0,1,\ldots,n-1 \quad ; \quad c_n^{(0)} = 0$$

for $k = 0,1,\ldots,n-1$

$$a_{k+1} = c_0^{(k-1)} / c_0^{(k)} \quad ; \quad b_{k+1} = c_{n-k}^{(k-1)} / c_{n-k-1}^{(k)}$$

$$c_i^{(k+1)} = c_{i+1}^{(k-1)} - a_{k+1}\, c_{i+1}^{(k)} - b_{k+1}\, c_i^{(k)} \quad , \quad i = 0,1,\ldots,n-2$$

9. Evaluation of a continued fraction

In the previous algorithms we obtained the reduced model in the form of a continued fraction. It is possible to transform it into other descriptions. If the parameters in the Cauer form continued fractions are given we can perform the corresponding Routh type algorithms in reverse order. This corresponds to the backward evaluation scheme for the continued fractions. We could also use the forward evaluation scheme or any other scheme. We obtain in each case the rational transfer function of the reduced system. The following references give algorithms for evaluation [CHA75, CHEN69, CHEb74, JOH81, KUMa78, KUMb78, KUM79, KUM80, PARTa75, PARTa76, PARTb76, PARTa79, PARTb79, PART80, RAOV75, RAOS74, SHIb71, SHIc74, SHId74, SHIa75, SIN79, SINd81].

10. Stability

An important aspect of linear systems is their stability. A necessary and sufficient condition for a system to be asymptotically stable (AS) is that the eigenvalues of the state transition matrix A

are in the left half plane (c.t.)
are inside the unit disc (d.t.).

Note that these eigenvalues are the zeros of the denominator of the rational function $C(zI-A)^{-1}B$.
If the poles of the reduced transfer function meet the conditions above, then the system is called bounded-input - bounded-output stable (BIBOS). AS implies BIBOS.

11. The Routh-Hurwitz, Schur-Cohn and related stability tests

Some references on the Routh-Hurwitz stability test are [BAR81, CHE66, CHAN74, KHAa81, KHAb81, KIM82, KRI72, KRI80, SHAc80, SHAd80, SHAc81, SINe81, TSO79, ZAK79].
It is known that the problem of root location for a polynomial is connected with continued fraction algorithms. The AS of a c.t. system can be checked using the well known

Theorem

A real polynomial $Q(z)$ of degree n has all its roots in the left half plane iff the Hurwitz alternant

$$r(z) = \frac{1}{\sqrt{z}} \frac{Q(\sqrt{z}) - Q(-\sqrt{z})}{Q(\sqrt{z}) + Q(-\sqrt{z})}$$

can be expanded as a finite continued fraction of the form

$$\cfrac{1}{b_1} + \cfrac{z}{b_2} + \cfrac{z}{b_3} + \ldots + \cfrac{z}{b_n} \quad \text{with } b_i > 0 \ , \ i = 1,2,\ldots,n \ . \tag{11.1}$$ □

The continued fraction (11.1) is equivalent with a continued fraction of the form

$$\cfrac{a_1}{1} + \cfrac{a_2 z}{1} + \cfrac{a_3 z}{1} + \ldots + \cfrac{a_n z}{1} \quad \text{with } a_i > 0 \ , \ i = 1,2,\ldots$$

The latter is called a Stieltjes fraction. Applications are found in [CHE66, SHAc80].
This may be confusing since the name Stieltjes fraction has also been attributed in system literature to a contraction of the first Cauer form (see (8.2)). For d.t. system stability we have to consider the location of the roots of a polynomial Q(z) of degree n with respect to the unit circle. Herefore define

$$r(z) = \frac{Q^{\star}(z)}{Q(z)} \quad \text{with} \quad Q^{\star}(z) = z^{n}\,\overline{Q(1/\bar{z})} \ .$$

Such a function can be expanded into a modified Cauer form continued fraction. It will have a special structure which is (equivalent with)

$$r(z) = \cfrac{1}{\bar{b}_1} + \cfrac{1-|b_1|^2}{b_1} + \cfrac{z}{\bar{b}_2} + \cfrac{1-|b_2|^2}{\bar{b}_2} + \ \ldots\ + \cfrac{z}{\bar{b}_{n+1}}$$

or

$$= b_1 + \cfrac{1-|b_1|^2}{\bar{b}_1} + \cfrac{1}{b_2 z} + \cfrac{(1-|b_2|^2)z}{b_2} + \ \ldots\ + \cfrac{1}{b_{n+1}z} \qquad (11.2)$$

with $|b_{n+1}| = 1$. We have the following

Theorem

Q(z) will have all its zeros inside the unit disc iff $|b_k| < 1$ for $k = 1,2,\ldots,n$ □

Because of the special structure, the Routh type algorithm to compute this expansion comes down to the simple Schur-Cohn recursions

$$\begin{array}{l} Q_n = Q(z) \\ \text{for } k = n\ ,\ n-1,\ldots,1 \\ \quad b_{n+1-k} = Q_k^{\star}(0)/Q_k(0) \\ \quad Q_{k-1} = Q_k - \bar{b}_{n+1-k}\, Q_k^{\star} \end{array}$$

where Q_k is a polynomial of degree k. See e.g. [JON82].

Other approaches in the literature [BISa82, DAVIS82, ISM83, SCHU76, STE77, SZC77] use a bilinear transform like $\zeta = (z+1)/(z-1)$ mapping the unit circle into a half plane, thus transforming the problem into a Routh-Hurwitz problem. The result is as follows :
Define for a real polynomial $Q(z)$

$$Q_e(z) = \frac{1}{2}[\,Q(z) + z^n\, Q(1/z)\,] \qquad (11.3)$$

$$Q_o = \frac{1}{2}[\,Q(z) - z^n\, Q(1/z)\,].$$

Then $Q = Q_e + Q_o$ while

$$\frac{Q_o}{Q_e} = \cfrac{1}{b_1\zeta} + \cfrac{1}{b_2\zeta} + \cfrac{1}{b_3\zeta} + \ldots + \cfrac{1}{b_n\zeta} \quad \text{with } \zeta = \frac{z+1}{z-1}\,.$$

We have the following stability theorem :

Theorem

$Q(z)$ has all its roots inside the unit circle if and only if the above continued fraction expansion has all $b_k > 0$, $k = 1,\ldots,n$ □

Several variants of the previous stability tests exist. They can be found in the literature E.g. [AND76, BAR74, DEL84].

12. Stable reduced models

An important drawback of the Padé approximants considered earlier to derive a reduced model is that it will not always give a stable approximant, even if the original system was stable. To overcome this, we could choose a reduced model which is forced to be stable. This means choose the denominator of the reduced transfer function to have stable zeros and compute a corresponding numerator of degree r-1 say such that it fits r time moments and/or Markov parameters. In this way we obtain what has been called by Brezinski a Padé type approximant. There are different methods to select the denominator. We mention the stability equation method, dominant pole selection, the Routh method and the Schwarz method.

12.1. Retention of dominant modes

Some references on this method are [HWAa81, LUd83, SHAc73, SHAa74, SHAa75, SHAb75, SHAe75, SHAb78, SHIc75, SHId75, SHIa76, SHIb76].
It is possible to select some of the poles of the original system transfer function and take them as the poles of the reduced model. Since we need not all the poles but only r of them, we can make use of the following well known

Theorem

Let $H(z)$ be meromorphic in $|z| < R$ with poles z_k, $k = 1,\ldots,s$ ordered such that $0 < |z_1| \leq |z_2| \leq \ldots \leq |z_s| < R = |z_{s+1}|$. Define $q(z) = \prod_{i=1}^{s} (1-z/z_i)$. Then the denominator q_s of the (m,s) Padé approximant of H, normalized such that it has a constant term equal to 1 will satisfy

$$q_s(z) = q(z)\ (1 + 0(|z_s/z_{s+1}|^m)) \quad \text{for} \quad m \to \infty \qquad \square$$

A similar result can be obtained for the Rutishauser qd-algorithm. More details on the latter can be found in [3]. Note that in this way we can find some of the poles closest to $z = 0$. In the same way we can find a number of poles near some other point, e.g. $z = \infty$. Once r of the poles are selected, we choose them to be the zeros of the denominator of the reduced system. It should also be mentioned that the speed of convergence of q_s to q depends on the ratio $|z_s/z_{s+1}|$. Thus, if the convergence is fast enough, we have detected a gap between $|z_s|$ and $|z_{s+1}|$, separating the poles near $z = 0$ which are dominant and those which are not.

12.2. Stability equation method

This method has been used in [BISb82, CHET79, CHETa80, CHETb80, CHETc80, HWAa81, PARTc82, SHAb81, THE82, THE83, WAN81].
Consider a c.t. system. Suppose the Hurwitz alternant for the denominator $Q(z)$ of the system transfer function is

$$r(z) = \frac{Q_o(z)}{Q_e(z)}$$

with

$$Q_o(z) = z^{-1/2}(Q(z^{1/2}) - Q(-z^{1/2})) = d_1 \prod_{i=1}^{k} (1+z/z_i) \quad , k = [\frac{n}{2}]$$

and

$$Q_e(z) = Q(z^{1/2}) + Q(-z^{1/2}) = d_0 \prod_{i=1}^{\ell} (1+z/p_i) \quad , \ell = [\frac{n-1}{2}]$$

where $0 < z_1 < p_1 < r_2 < p_2 \quad \ldots \quad .$ (12.1)

The stability equation method selects some of the z_i and p_i to obtain an Hurwitz alternant for the reduced system. Which of the z_i and p_i are kept depends on where we want the approximation to be good. e.g. at $z = 0$, $z = \infty$ or both. The selection is also subject to a condition like (12.1). Once this decision has been made, we know the Hurwitz alternant and hence the denominator of the stable reduced system transfer function. This method has the obvious drawback that the polynomials Q_o and Q_e have to be factored, but their degree is about one half of the degree of the original polynomial Q.
Moreover, these polynomials have negative zeros with an interlacing property (12.1). This makes it easier to compute them. For d.t. systems the Schur-Cohn approach does not seem to be directly applicable in a similar way, but the modified approach with the transformation $\zeta = (z+1)/(z-1)$ gives similar results. E.g. the zeros of Q_e and Q_o defined in (11.3) ly on the unit circle and interlace. [BISb82].

12.3. The Routh-Padé and Schwarz method

These methods have been very popular. The Routh-Padé method was used in [APP78, APP79, ASa82, BISb79, BIS81, HUTa74, HUTb74, HUT75, HUT77, HWAa81, HWAb82, JOH79, KRI76, KRIa78, KRIb78, LAN78, LUa83, MAR79, PALa79, PALa80, PALd80, PARTc79, SHAc75, SHAa78, SHAb79, SHAa80, SHAb80, SHAb81, SINH80] and the Schwarz method is treated in [BAD80, DAV76, LALa75, LUa83, RAOb78, RAOa81, RAO83, RAOV80, SHAc78].
Another variant using the Hurwitz alternant for c.t. system is directly based on the stability theorem. Suppose we compute by the Routh algorithm the continued fraction for the Hurwitz alternant

$$\frac{Q_e}{Q_o} = \cfrac{1}{\left\lceil b_1 \right.} + \cfrac{z}{\left\lceil b_2 \right.} + \cfrac{z}{\left\lceil b_3 \right.} + \ldots \quad \text{with} \quad b_i > 0.$$

Then an approximation for this alternant can be obtained by taking only the first r terms in this expansion. Since the b_i are positive, this truncated continued fraction will also be the expansion of a Hurwitz alternant for a stable polynomial of degree r. This again gives a denominator for a stable reduced system. Note that from any continued fraction of the above form we can construct a Hurwitz polynomial [SINb81]. Thus we could also invert the order of the coefficients. I.e. apply the above procedure for $Q^{\star}(z) = z^n Q(1/z)$ which will be stable if Q(z) is. This gives a reduced polynomial $Q_r^{\star}(z)$ from which $Q_r(z) = z^r\, Q_r^{\star}(1/z)$ is found, which is in some sense a stable approximant for Q(z). A stable approximant could also be constructed ty taking the tail of the above continued fraction. This would save the work of inverting the Routh table to obtain approximants for Q_e and Q_o since these approximants directly appear in the Routh table of the original system. This is exactly what is done in the Schwarz method. As with the stability equation method, we approximate the Hurwitz alternant rather than the denominator itself. It is then not completely clear in what sense the denominator Q will be approximated by Q_r. Non dominant modes could be approximated by accident [SHAa81, MARb83, PALa83]. Instead of Padé like approximation, we could also use rational interpolation at different points like in [APP77]. The Routh and Schwarz methods can also be adapted for d.t. systems, either by the discrete Routh approach or by the Schur-Cohn approach. The form of the continued fraction for d.t. systems will be different but the principle is exactly the same.

12.4. Evaluation of the numerator

In the previous methods we saw how a stable denominator Q_r could be found. The corresponding numerator is found from the Padé requirements. These can be imposed in $z = \infty$ to match the Markov parameters, in $z = 0$ (c.t.) or $z = 1$ (d.t.) to match the time moments or in some other point or a combination of them. Suppose e.g. that you want to fit the first r time moments of a c.t. system. Then the numerator is defined to be the first r terms in the expansion

$$H(z)\, Q_r(z) = \sum_0^{\infty} c_k\, z^k.$$

The numbers c_k can be simply computed if we know the expansion of H(z) near z = 0, i.e. if we know the time moments. In many cases we know H(z) as a ratio of two polynomials. In such a situation we can compute the numerator by some inversion of the Routh algorithm. Compute by the Routh algorithm the first r coefficients b_k, k = 1,2,...,r of the second Cauer form expansion for H. If the reduced system H_r has to match the first r time moments, then the first r coefficients b_k in its second Cauer form expansion must be the same as for the original system. Thus we know for the Routh table of the reduced system what the coefficients b_k, k = 1,2,...,r must be and we also know the first row (the reduced denominator). It is then possible to reconstruct the complete Routh table for H_r. The numerator appears as the second row in the table. The same principle can be applied to other Cauer form expansions to obtain Markov parameter matching or mixed approximations. More details are found in the cited literature. In the Padé approximation methods we obtained the reduced approximant as a truncated continued fraction expansion. The Routh-Padé approximant can also be found in closed form as a truncation of some special type of expansion. The principle is the following : We can always find a formal expansion

$$H = \frac{P_1 + P_2}{Q_0 + Q_1} = \frac{Q_1}{Q_0 + Q_1} \left(\frac{P_1 + P_2}{Q_1}\right) = F_1 \left(a_1 + \frac{P_2 + P_3}{Q_1}\right)$$

where $F_1 = Q_1/(Q_0 + Q_1)$ and $P_3 = P_1 - a_1 Q_1$ for some a_1,

$$H = F_1 \left(a_1 + F_2\left(a_2 + \frac{P_3 + P_4}{Q_2}\right)\right)$$

where $F_2 = Q_2/Q_1$ and $P_4 = P_2 - a_2 Q_2$ for some Q_2 and some a_2,

$$H = \ldots$$

$$= \sum_k a_k \prod_{j=1}^{k} F_j$$

Suppose H(z) is the rational transfer function of the original system and the splitting P_1, P_2 and Q_0, Q_1 are the odd and even parts of numerator and denominator. An expansion of the above type for n odd can be obtained by choosing for Q_k the polynomials appearing in the Routh

algorithm for Q_1/Q_0 constructing an expansion around $z = \infty$. I.e.

$$\frac{Q_1}{Q_0} = \cfrac{1}{b_1 z} + \cfrac{1}{b_2 z} + \ldots + \cfrac{1}{b_n z} \quad \text{where } b_k = \lim_{z\to\infty} \frac{Q_{k-1}(z)}{z\, Q_k(z)}, \qquad (12.2)$$

and $Q_{k+1} = Q_{k-1} - b_k z Q_k$.

Then

$$F_1 = \cfrac{1}{1+b_1 z} + \cfrac{1}{b_2 z} + \ldots + \cfrac{1}{b_n z}$$

and

$$F_k = \cfrac{1}{b_k z} + \cfrac{1}{b_{k+1} z} + \ldots + \cfrac{1}{b_n z} \quad \text{for} \quad 2 \leq k \leq n.$$

For n even the role of Q_0 and Q_1 have to be interchanged.
The a_k are chosen as $\lim_{z\to\infty} P_k(z)/Q_k(z)$.
If the original system has degree n, then there will be n terms in the expansion since the degrees of the polynomials Q_k decrease in each step. The reduced Routh approximant of degree r is obtained by taking the first r terms. [HUTa74, HUTb74, HUT75, KRI76, RAOa78].
For further reference we shall call this expansion the ab expansion. The Schwarz approximant as originally proposed is obtained by essentially taking the last r terms [LUa83]. In this form however it has not the moment matching property. The ab expansion has several advantages since we obtain the impulse response energy of the r th approximant by

$$\|h_r\|^2 = \int_0^\infty h_r^2(t)dt = \sum_{k=1}^{r} \frac{a_k^2}{2\, b_k}. \qquad (b_k > 0 \text{ for stable systems})$$

We thus have

$$0 \leq \|h_1\| \leq \|h_2\| \leq \ldots \leq \|h_n\| = \|h\|.$$

If a certain amount of $\|h\|$ is represented by the reduced system $\|h_r\|$ we can consider the approximant to be good. This gives a criterion to find r.
Another advantage is that we have a flow graph representation of H_r from which a state space description can be read off. The complete algorithm can be translated into a state space algorithm [RAOa78].

12.5. Some variants

Instead of fixing the denominator of the reduced system completely we can fix only part of it. In this way more degrees of freedom are left to match Markov parameters and/or time moments. E.g. if you fix s of the zeros of the reduced order denominator, then the last s Padé equations are replaced by the conditions $Q_r(z_i) = 0$, $i = 1,2,\ldots,s$ if Q_r is the reduced order denominator and z_i the selected zeros for it. [SHAb75]. Another variant consists in allowing the denominator of the stable reduced denominator to be perturbed such that it remains stable [SINa81, SINc81]. Systematic procedures exist that introduce some parameters in the denominator. These extra degrees of freedom may again be used to fit some extra time moments or Markov parameters [LEPc82] or if the system is unstable to make it stable if possible while loosing as few of the interpolation properties as possible. This idea is misused [LU84] in [CHU75,CHU76] . Other methods to obtain a stable donominator exist. It may be e.g. be obtained by differentiation [LEPa83].

13. Block diagram for a continued fraction

Consider the following relation

$$Y = \left(\frac{a_1}{|b_1} + \frac{a_2}{|b_2} + \frac{a_3}{|b_3} + \ldots\right)U.$$

This can be decomposed as follows. Define $X_{-1} = U$ and $X_0 = Y$ and recursively

$$X_k = a_k X_{k-2} - b_k X_{k-1} \quad , \quad k \geqslant 1.$$

Then we obtain the relation

$$X_k = \left(\frac{a_{k+1}}{b_{k+1}} + \frac{a_{k+2}}{b_{k+2}} + \ldots\right) X_{k-1} \quad , \quad k \geq 1.$$

These can be brought into a block diagram of fig. 1 as can easily be verified [MIT72, MIT73, MIT74, MIT75].

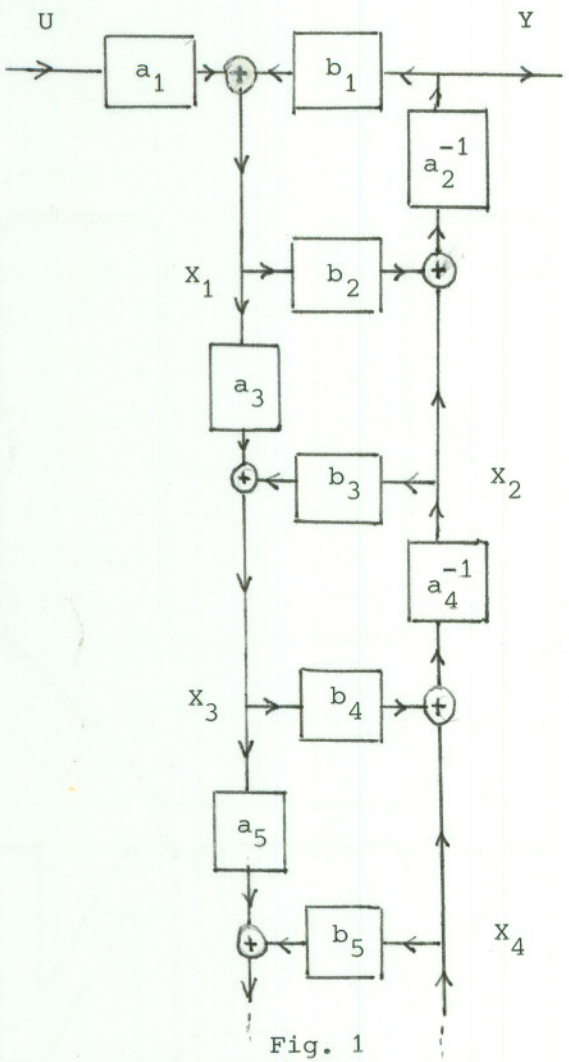

Fig. 1

Modifications of this scheme are possible. They often correspond to transformations of the continued fraction. Problems of computability may arise for some of the schemes if they are implemented as a digital or an analog network. In this context see [BAS83, BEL68, CAU26, CAU54, CHA74, CHEb74, CICa80, CICb80, CICc80, CIC81, CIC82, COO77, HAS76, KHA78, MIT72, MITa73, MITb73, MIT74, MIT75, MOR78, NEU82, NEW66, ORC70, SHIa81, SHIc81, SOB72, SOB73, SOB75, SU71, WEI62, YAN83].

14. State space formulations

By writing the third Cauer form continued fraction as

$$\cfrac{1}{h_1+k_1\,\zeta} + \cfrac{1}{h_2/\zeta+k_2} + \cfrac{1}{h_3+k_3\,\zeta} + \cfrac{1}{h_4/\zeta+k_4} + \ldots + \cfrac{1}{h_n/\zeta+k_n} \quad , \; n \text{ even,}$$

and by carefully choosing some state variables we can derive the following frequency domain equations from the corresponding block diagram (capitals U, X, Y correspond to the frequency transforms of the corresponding time functions u, x, y). [HWa82, HWA83, KHA80, SHIb74, SHIb75].

$$(\zeta^2 A_1 + \zeta A_2 + A_3)X = b\,U \,, \tag{14.1}$$

$$Y = (C_h + \zeta C_k)X \,,$$

with

$$A_1 = K_o K_e \,,$$

$$A_2 = I + K_o H_e + H_o K_e \,,$$

$$A_3 = H_o H_e \,,$$

$$K_o = \begin{bmatrix} k_1 & & & \\ k_1 & k_3 & 0 & \\ \vdots & & & \\ k_1 & k_3 & & k_{n-1} \end{bmatrix} \,, \qquad K_e = \begin{bmatrix} k_2 & k_4 & k_6 & \ldots & k_n \\ & k_4 & k_6 & & \\ & & k_6 & & \\ 0 & & & & k_n \end{bmatrix} .$$

H_o and H_e are similar with k replaced by h.

$$b^t = [1 \quad 1 \ \ldots \ 1]$$

$$C_h = [h_2 \quad h_4 \ \ldots \ h_n]$$

$$C_k = [k_2 \quad k_4 \ \ldots \ k_n]$$

The previous results are for n even but for n odd similar results may be obtained. As an example consider a c.t. system ($\zeta = z$). Then these equations can be translated into time domain state space equations as follows

$$\dot{x} = A\,x + B\,u$$
$$y = C\,x \qquad (14.1)$$

with

$$A = \begin{bmatrix} 0 & I \\ -A_1^{-1} A_3 & -A_1^{-1} A_2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ A_1^{-1} b \end{bmatrix},$$

$$C = [C_h \quad C_k],$$

$$x^T = [x_1^T \quad x_2^T] \; ; \; x_1 = x \; ; \; x_2 = \dot{x}.$$

This realisation is minimal provided all k_i and h_i are nonzero. The first and second Cauer forms appear as special cases in the previous derivations [AGA76, CHE68, CHE69, CHEa74, LAM78, PARTd82, PARTa83, SHIb74, SHIa75, SHIb75, SHI77, SHIa81]. For discrete time systems we have to set $\zeta = z-1$ and a similar derivation is possible. Note that the reduced order system is obtained by taking only the first r rows and/or the first r columns in A_i $(i = 1,2,3)$, b, C_h and C_k.
The previous algorithms which derived the Cauer form continued fractions from the rational transfer functions can be seen as algorithms performing some equivalence transformation of the state space equations bringing the phase variable canonical form into the corresponding Cauer canonical form. The transformation matrix contains rows of the corresponding Routh table [BAN84, HWAa82, HWA83]. If the system is given by its state space description which is not in phase variable canonical form it is possible to construct the transformation, bringing it into its Cauer canonical form directly. Once the state space description in a canonical form is obtained, the state space description of the reduced model is obtained by deleting the last n - r states. A reduced model

can be obtained in principle by some appropriate aggregation law transforming the states by

$$v = P\,x$$

with

$$P \in \mathbb{C}^{r \times n} .$$

[COM79, DAV79, HICa76, HICb76, HICc76, HIC79, HICa80, HICb80, LAM78, RAOb81]. This gives state space algorithms closely related to the previous Padé algorithms but where the connection with continued fractions and Padé approximants has somewhat been obscured. As an example we consider for continuous time description (14.1) the Schwarz and Routh canonical form. They can both be formulated in terms of the coefficients of the ab expansion given in section 12.4. For the Schwarz canonical form they are given by

$$B^T = [0 \ \ldots \ 0 \quad 1] \quad , \quad C = [c_1 \quad c_2 \ \ldots \ c_n] ,$$

$$A = \text{tridiag} \begin{bmatrix} 1 & 1 & \cdots & & 1 \\ 0 & 0 & \cdots & & 0 \ -(b_1)^{-1} \\ -(b_n\, b_{n-1})^{-1} & -(b_{n-1}\, b_{n-2})^{-1} & \ldots & -(b_1\, b_2)^{-1} & \end{bmatrix}$$

with $c_k = a_k/(b_1\, b_2 \ \ldots \ b_k)$, [AND76, LUa83, SMIc78],

and for the Routh canonical form they are given by [HUT75, LUa83]

$$B^T = [1/b_1 \quad 0 \ \ldots \ 0] \quad , \quad C = [a_1 \ a_2 \ \ldots \ a_n] ,$$

$$A = \text{tridiag} \begin{bmatrix} -(b_1)^{-1} & -(b_2)^{-1} & \ldots & -(b_{n-1})^{-1} \\ -(b_1)^{-1} & 0 & \ldots & 0 \\ (b_2)^{-1} & (b_3)^{-1} & \ldots & (b_n)^{-1} \end{bmatrix} .$$

For d.t. systems such canonical forms may also be obtained. See e.g. [AND76, BAD80].

15. Minimal phase transfer functions

In some applications it can be desirable that the zeros of the transfer functions satisfy conditions like the poles of a stable transfer function. The so called minimal phase conditions.
A transfer function (and the corresponding system) is said to be minimal phase if the finite zeros are in the left half plane (c.t.) or inside the unit disc (d.t.). It is clear that what is said before about the stability of a system can be easily transformed into corresponding minimal phase statements. [KRI78].

16. Frequency response and Power spectrum

Let $H(z)$ be the transfer function of a system. Consider only information given on the boundary ∂S of the stability region S, i.e. S is the left half plane and ∂S is the imaginary axis $\partial S = \{z \mid z=i\omega\}$ for c.t. and S is the unit disc and ∂S the unit circle $\partial S = \{z \mid z=\exp i\omega\}$ for d.t. Thus what we know is

$$F(\omega) = H(z)\Big|_{z \in \partial S} .$$

$F(\omega)$ is called the frequency response of the system. Given the gain function $|F(\omega)|$, it is possible to reconstruct a stable minimal phase transfer function having the same modulus on the unit circle [5].
The power spectrum is $|F(\omega)|^2$. If we suppose that the transfer function has real coefficients, then the power spectrum is given by $F(\omega)\ F(-\omega)$. Thus

$$|F(\omega)|^2 = R(z)\Big|_{z \in \partial S} = H(z)\ H(z^{\star})\Big|_{z \in \partial S}$$

where $z^{\star}$ means the reflection of z in ∂S :

$$z^{\star} = 1/z \text{ (d.t.)} \qquad \text{and} \qquad z^{\star} = -z \text{ (c.t.)}.$$

$R(z)$ is a rational function of the same degree as $H(z)$ but in the variable $\zeta = \frac{1}{2}(z+z^{-1})$ (d.t.) or $\zeta = z^2$ (c.t.). The zeros and poles exhibit a double symmetry. Indeed, there is symmetry with respect to the real axis and there is symmetry with respect to a reflection in ∂S by the transformation $z \to z^{\star}$.

By solving a spectral factorization problem one can always recover a stable, minimal phase transfer function from R(z). The method of model reduction now consists in finding a reduced rational function R_r, approximating R. If R_r is factorizable, one of the factors can always been taken to be minimal phase and stable. The problem now is that R_r need not necessarily be a squared magnitude function on ∂S so that factorization is impossible. A problem which has not always been recognized.

17. Power spectrum approximation by Chebyshev-Padé methods

Chebyshev-Padé techniques are most popular in this application [BISa79,CAL76,CHE76,DAR52,DEB73,LAN80,LEPc83,LEP84] although ordinary Padé [HSI72] and interpolation in several points [LEPa82] may also be used.
Two types of Chebyshev-Padé approximations exist.
The Maehly method approximates a function r(x), $x \in [-1,1]$ by a rational function T/N such that

$$r\ N-T = \sum_{p}^{\infty} c_k\ T_k(x)$$

where T_k are the Chebyshev polynomials and p is as high as possible. The Clenshaw-Lord-Gragg-Johnson type of approximation finds T/N such that

$$r - T/N = \sum_{p}^{\infty} \tilde{c}_k\ T_k(x).$$

This is a simple extension of the Padé technique where now the Chebyshev polynomials replace the powers of x. But unlike in the classical case, the above methods are not equivalent anymore.
In the latter method, a Faber transformation $w = x + (x^2-1)^{1/2}$ transforms the interval [-1,1] to the unit circle so that Chebyshev series in x transform into ordinary Laurent series in w. The method to compute the Chebyshev-Padé approximants w.r.t.x is then essentially the method used for ordinary Padé approximants w.r.t.w.
For the d.t. case, the power spectrum is a rational function where the terms in numerator and denominator can be grouped to form terms in $\frac{1}{2}(z^k+z^{-k})$ which is $T_k(x)$ if $z = \exp i\omega = x+(x^2-1)^{1/2}$. So that the Faber transform need not be explicit. For c.t. systems an appropriate transformation brings the power system into a similar form.

The ultimate problem that has to be solved is to find a rational approximant of a function R(z) having the form

$$R(z) = H(z)\ H(1/z) = \sum_{-\infty}^{\infty} r_k\ z^k \qquad \text{with } r_k = r_{-k}.$$

The practical method to solve this is the Laurent-Padé approach. We have to split up R(z) as

$$R(z) = Z(z) + Z(1/z) \text{ with } Z(z) = \frac{1}{2} r_0 + \sum_1^{\infty} r_k\ z^k.$$

Suppose we want to find an approximant with numerator degree s and denominator degree r. We then have to construct a rational function P/Q with numerator degree $l = \max\{s,r\}$ and denominator degree r such that the coefficients of z^k in the formal product R(z)Q(z) vanish for $k = s+1,\ldots,r+s$. This defines the denominator Q(z) up to a multiplicative constant by a linear system of equations. The numerator is then found as the first l+1 terms in the product Z(z)Q(z). With P/Q we construct a reduced approximant R_r for R by

$$R_r = \frac{P(z)Q(1/z)+P(1/z)Q(z)}{Q(z)\ .\ Q(1/z)}\ . \qquad (17.1)$$

The numerator will be a Laurent polynomial of degree s. As you can see from this, the denominator can always be factorized in stable-unstable factors. The problem that arises is that the numerator, although it is symmetric in z and 1/z, need not be factorizable in minimal phase-maximal phase factors because it may have zeros on the unit circle of odd multiplicity. To overcome this realizability problem, we could follow an approach similar to the Routh-Padé method. I.e. we could force the numerator of R_r to be of the form P(z) P(1/z) with $P(1/z) = c\prod(1-\alpha_k/z)$, $|\alpha_k| < 1$, so that it is factorizable. A possible choice is to take all $\alpha_k = 0$. The well known Levinson and Schur algorithms are recursive methods to find such approximants. The final approximant for H is then of the so called auto-regressive form i.e., a constant divided by a polynomial. [BUL80,BUL82,BULa82, CYB82,JON82].

This type of approximants has several interesting properties. It can be shown from the positivity of R(z) on $|z| = 1$ that

$$\Gamma(z) = \frac{1}{z} \frac{Z(z) - \frac{1}{2} r_0}{Z(z) + \frac{1}{2} r_0}$$

is a Schur function. This means that it is analytic inside the closed unit disc and bounded by 1. The continued fraction associated with these algorithms is an expansion of $Z(z)$ or $\Gamma(z)$ and they are of a form that is much like the continued fraction (11.2). [SCHU76] The parameters b_k appearing in (11.2) are called reflection coefficients or partial correlation coefficients. The term reflection coefficient comes from scattering theory. It is much used in the construction of ladder or lattice networks realizing such a transfer function [BEL68,DEL84]. It also follows from the properties of $R(z)$ that these reflection coefficients are bounded by 1 in modulus and this implies that the polynomial Q of (17.1) has all its zeros in the unit disc so that in this case there is no need to redistribute the poles to obtain a stable-unstable factorization. Other properties can be attributed to these approximants. They are e.g. weighted least squares approximants and maximum entropy approximants.

If not all the transmission zeros α_k are located at the origin one can obtain a general autoregressive-moving average (ARMA) type of transfer function. The Laurent-Padé approximant is then modified to a multipoint version interpolating at the points α_k. The algorithm to obtain these approximants is an interpolatory version of the Schur algorithm and it is known by the name of Nevanlinna-Pick algorithm [DEL81]. The choice of the interpolation points α_k is more or less arbitrary. The best choice would be to take the dominant transmission zeros of the original transfer function i.e. those closest to the unit circle. They can be found from the qd table associated with the Fourier series of $R(z)$. [BULa83,BULb83]

The Levinson and Schur algorithm work without problems if $R(z)$ is strictly positive on the unit circle. If this is not so some modifications are in order [CHUI82]. A related Padé type approximant is given in [CHUI79,CHEN80].

It is possible to obtain a continued fraction expansion for a rational function expressed in terms of orthogonal polynomials [MARO80].

There is an extensive literature on the Levinson-Schur algorithms in a linear predictive filtering framework which we did not include in our survey. We give only its relation with Chebyshev-Padé approximation.

18. Simultaneous approximation of impulse response and power spectrum

In [MUL76,HAST77,INOa83,INOb83] an approximant is constructed for a d.t. system which is partially a Padé approximant because the first m+1 Markov parameters(impulse response data) and partially a Laurent-Padé approximant because the first n+1 autocorrelation parameters (power spectrum data) are fitted.
More precisely. Suppose the transfer function is

$$H(z) = \sum_{0}^{\infty} h_k z^{-k}$$

and

$$H(z)H(1/z) = \sum_{-\infty}^{\infty} r_k z^{k}, \qquad r_{-k} = r_k.$$

The method constructs an approximant H_r of the form $\dfrac{c_0+c_1z^{-1}+\ldots+c_mz^{-m}}{1+d_1z^{-1}+\ldots+d_nz^{-n}}$ where

$$H_r(z) = \sum_{0}^{\infty} \hat{h}_k z^{-k},$$

$$H_r(z)H_r(1/z) = \sum_{-\infty}^{\infty} \hat{r}_k z^{k}, \qquad \hat{r}_{-k} = \hat{r}_k,$$

which satisfies the conditions $\hat{h}_k = h_k$, $k = 0,1,\ldots,m$ and $\hat{r}_k = r_k$, $k = 0,1,\ldots,n$.
A computational procedure is presented which resembles the Levinson algorithm. The approximant is shown to be stable.

19. Least squares approximation and other types of approximation

In least squares approximation for H(z) one tries to minimize a least squares error. E.g. for a d.t. system, H_r has to be found such that

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{i\omega}) - H_r(e^{i\omega})|^2 \, d\omega$$

or

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{i\omega}) / H_r(e^{i\omega}) - 1|^2 \, d\omega$$

or some variant of this is minimized. [CHUI77].

This is a nonlinear optimization problem which requires some starting values. As initial guess to the solution one can take e.g. a Padé approximant [BROP73].
Least squares approximation can also be formulated in the time domain. Then one wants to approximate

$$H(z) = \sum_0^\infty h_k z^{-k} \qquad \text{by} \qquad H_r(z) = \sum_0^\infty \hat{h}_k z^{-k}$$

by minimizing

$$\sum_0^\infty w_k (h_k - \hat{h}_k)^2$$

where w_k are some weights. In the Padé approximation problem one sets $w_k = 1$ for $k = 0,1,\ldots,m+n$ and $w_k = 0$ for $k > m+n$ which defines a semi-norm on the impulse response sequences. With an exponentially decreasing weight e.g. one obtains something which is somewhere between Padé and least squares [BROP74,HAST77].
A Padé approximant could also be chosen as a starting value to construct approximants that meets certain properties of the original transfer function [SHIc76,SHI79,SHIb80,SIN82].

20. Non-normal Padé tables

Until now we supposed that all the different types of Padé approximants existed and that they could be computed by the algorithms mentioned, which were in most cases of Routh-type. It is well known that in general the table of Padé approximants posesses a so called block structure. This means that there are square blocks of equal entries in such a table. If a Routh type algorithm has to pass such a block it will break down because a zero division will appear. An extra problem that appears in a non-normal Padé table is the problem of minimality. Indeed reducible approximants may arise. Since it is obviously most interesting to have the reduced form, the recursive algorithms should be carefully formulated such that they produce the reduced (i.e. the minimal) approximants. Moreover, the approximant need not be unique, even if it is minimal. The Routh-type algorithms have to be modified and become somewhat more involved. We have to use here the Euclidean algorithm, the Berlekamp-Massey algorithm or variants thereof. In [GRA83] an excellent treatment of this problem is given which extends our section 7.2 and the

corresponding state space description to the non-normal situation. The continued fraction (7.5) takes now the form

$$\frac{1}{|b_1(z)} + \frac{1}{|b_2(z)} + \frac{1}{|b_3(z)} + \cdots$$

where the $b_k(z)$ are polynomials in general.
Other references on the non-normal Routh algorithm and on stability checking algorithms in a non-normal situation for SISO systems are [DEL84,KAL79,KHAa81,KHAb81,SHO83,WRI73,YEU83]. In [DEL83] a non-normal Levinson algorithm is derived.

21. MIMO systems

If the system is multi-input-multi-output (MIMO), the situation is even much more complicated. In a situation where we assume that normality conditions hold, there doesn't seem to be a big problem. The theory introduced before can be generalized to the matrix case. If the number of inputs is not equal to the number of outputs, so that the matrices are non-square there may be some trouble in adapting the continued fraction interpretation. In a non-normal situation, which is the most probable for the matrix case, it is not a trivial matter to adapt the scalar theory. To give a detailed explanation of the theory would bring us to far and is not possible in this brief survey. We shall only give an enumeration of the references we included in our bibliography.
The second matrix Cauer form and fitting of time moments is considered in [BOSc80,CHEa74,DEC76,GOL81,SHIa74,SHId74,SHIa75,SHIb75, SHIe75,SHI77,SHIa78,SHIa81,SHIb81,SHIH78,ZAK73].
The first Cauer form is related to the problem of minimal partial realization. There is a very large number of papers on this topic. Many of them are based on an analysis of the Hankel matrix of Markov parameters. Most of them are not included in our bibliography. We have only selected some of them where the relation with Padé approximation is more or less obvious. Algorithms of Euclid or Berlekamp-Massey type are considered in [DIC74,FUH83,HICa76,PARTc75, TSO76]. The Cauer first form continued fraction features in [DEC76, GOL81,KHAb75,SHAf75,SHId74,SHIb75]. The modified and mixed Cauer forms are used in [BISc82,BIS84,DEC76,KHA80,LALb75,LALb76,MAI76, MARa83,PARTb78,PARTc83,SHAa76,SHId74,SHIb75,SHI77,SHIc80].

On evaluation of continued fractions see [KUM79,PARTa76,SHId74,SHIa75]. For the computation of stable approximants we have papers on dominant mode retention [SHAb75,SHAe75,SHIc75,SHId75,SHIa76,SHIb76], stability equations [CHETa80,CHETc80,THE83,WAN81], Routh-Padé [APP78,PALa80,PALd80, SHAc75,SHAb80,SHAb81,SINH80] and Schwarz-Padé [SHIc78,SHId78].
On the networks associated with continued fractions you find information in [BAS83,COO77,MOR78]. The method of Mullis and Roberts has been generalized to the matrix case in [INOb83].

22. Two dimensional systems

In some applications, the input and output of a system may not be a function of time but they can be functions of two space coordinates like e.g. in picture processing problems. In this case one speaks of two dimensional (2-D) or in general multidimensional systems. Many definitions of Padé approximation, extending the classical concept are possible in this case. Much depends on which powers of x and y are allowed in numerator and denominator of the approximant and on the coefficients of which powers are matched by the approximant. Two dimensional Padé approximants and branched continued fractions are used in [BAS81,BOSa80,BOSb80,CHEN80,CICa80,CICb80,CIC82,GAR81, HWA84,LEV79,MIT75,PARa80,PARb80,RAOS76,RAOS77].

23. Conclusion

We have illustrated that all kinds of Padé approximants and continued fractions have been used in solving the problem of model reduction. These ideas were introduced in the simplest possible situation of a scalar transfer function that is supposed to satisfy certain normality conditions. In a more general situation of non-normal, non square MIMO transfer function, the correct extension of these ideas is certainly not trivial.
It should be noted that we concentrated on Padé type approximants and did not include other important model reduction methods [2]. Note, further that the bibliography is certainly not complete in all respects. E.g. the partial minimal realization problem in all its generality is only mentioned briefly. This is worth a survey on its own and is not included in extenso in the list of references. Neither is a huge number of papers on the Levinson/Schur type algorithms, which would probably double the number of entries in the list.

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